ARENS REGULARITY AND DERIVATIONS OF HILBERT MODULES WITH THE CERTAIN PRODUCT

A. SAHLEH * AND L. NAJARPISHEH

Abstract. Let $A$ be a $C^*$-algebra and $E$ be a left Hilbert $A$-module. In this paper we define a product on $E$ that making it into a Banach algebra and show that under the certain conditions $E$ is Arens regular. We also study the relationship between derivations of $A$ and $E$.

1. Introduction and preliminaries

The notion of Hilbert $C^*$-module is a natural generalization that of Hilbert space arising by replacing of the field of scalars $\mathbb{C}$ by a $C^*$-algebra. For commutative $C^*$-algebras, such generalization was described for the first time in the work of I. Kaplansky [6] and the general theory of Hilbert $C^*$-modules appeared in the basic papers of W. L. Paschke [10] and M. A. Rieffel [11]. Let us recall these notions with more details.

Let $A$ be a $C^*$-algebra and $E$ be a linear space which is a left $A$-module with a compatible scalar multiplication. The space $E$ is called a left pre-Hilbert $A$-module if there exists an $A$-valued inner product $E\langle \cdot, \cdot \rangle : E \times E \rightarrow A$ with the following properties:

(i) $E\langle x, x \rangle \geq 0$ and $E\langle x, x \rangle = 0$ if and only if $x = 0$;
(ii) $E\langle \lambda x + y, z \rangle = \lambda E\langle x, z \rangle + E\langle y, z \rangle$;
(iii) $E\langle a.x, y \rangle = a E\langle x, y \rangle$;
(iv) $E\langle x, y \rangle^* = E\langle y, x \rangle$ for all $x, y, z \in E, a \in A, \lambda \in \mathbb{C}$.

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*Corresponding author.
From the validity of a useful version of the classical Cauchy-Schwartz inequality it follows that \( ||x|| = \|\mu(x,x)\|^\frac{1}{2} \) is a norm on \( E \) making it into a normed left \( A \)-module [7]. The left pre-Hilbert module \( E \) is called left Hilbert \( A \)-module if it is complete with respect to the above norm. One interesting example of left Hilbert \( C^* \)-modules is any \( C^* \)-algebra \( A \) as a left Hilbert \( A \)-module via \( _A(a,b) = ab^*(a,b \in A) \).

The left Hilbert \( A \)-module \( E \) is called full if the closed linear span \( \langle E, E \rangle \) of all elements of the form \( \mu(x,y) \) \((x,y \in E)\) is equal to \( A \). Likewise, a right Hilbert \( A \)-module with an \( A \)-valued inner product \( \langle \cdot , \cdot \rangle_E \) can be defined. The reader is referred to [7] for more details on Hilbert \( C^* \)-modules.

For a normed space \( X \), we denote by \( X' \) the topological dual of \( X \). Now, let \( X, Y \) and \( Z \) be normed spaces and let \( f : X \times Y \to Z \) be a bounded bilinear map. In [2], R. Arens showed that \( f \) has two natural but different extensions \( f'' \) and \( f^{'''r} \) from \( X'' \times Y'' \) to \( Z'' \). The adjoint \( f' : Z' \times X \to Y' \) of \( f \) is defined by \( < f'(z',x), y > \geq < z', f(x,y) > \) for all \( x \in X, y \in Y, z' \in Z' \), which is also a bounded bilinear map. By setting \( f'' = (f')' \) and continuing in this way, the mapping \( f'' : Y'' \times Z' \to X' \), \( f''' : Y''' \times Z' \to X' \) may be defined similarly.

We also denote by \( f' \) the reverse map of \( f \), that is, the bounded bilinear map \( f' : Y \times X \to Z \) defined by \( f'(y,x) = f(x,y) \) for all \( x \in X, y \in Y \), and it may be extended as above to \( f''' : X'' \times Y'' \to Z'' \).

The map \( f \) is called Arens regular when the equality \( f''' = f^{'''r} \) holds. Two natural extensions of the multiplication map \( \pi : X \times X \to X \) of a Banach algebra \( X \), \( \pi'' \) and \( \pi^{'''r} \), are actually the so-called first and second Arens products, which will be denoted by \( \Box \) and \( \Diamond \), respectively. The Banach algebra \( X \) is said to be Arens regular if the multiplication map \( \pi \) is Arens regular. For example \( L^1(G) \) is Arens regular if and only if \( G \) is finite [13].

A derivation of an algebra \( A \) is a linear mapping \( D \) from \( A \) into itself such that \( D(ab) = D(a)b + aD(b) \) for all \( a, b \in A \). For a fixed \( b \in A \), the mapping \( a \mapsto ba - ab \) is clearly a derivation, which is called an inner derivation implemented by \( b \).

Throughout this paper \( A \) denotes a \( C^* \)-algebra. We recall that every Hilbert module is a Banach space but the algebraic properties of Hilbert modules are our interesting subject. So in this note we utilize the \( A \)-valued inner product of Hilbert module \( E \) and define a product on \( E \) that making it into a Banach algebra. Our goal is finding the conditions under which \( E \) is Arens regular. We also study derivations of \( E \) and give some conditions under which innerness of derivations on \( A \) implies the innerness of derivations on \( E \) and vice-versa. Finally we
give a necessary and sufficient condition under which every derivation of \( C(X, H) \) is zero.

2. Arens regularity of Hilbert modules

In this section we introduce a product on a left Hilbert \( A \)-module that making it into a Banach algebra and study Arens regularity of this Banach algebra.

Let \( E \) be a left Hilbert \( A \)-module, and let \( e \) be an arbitrary element in \( E \) with \( ||e|| = 1 \). Then by a direct calculation the map \( \pi_e : E \times E \rightarrow E \) defined by \( \pi_e(x, y) = \langle x, e \rangle y \) is a product on \( E \) that making it into a Banach algebra. We denote this Banach algebra by \((E, \pi_e)\).

Example 2.1. Let \( X \) be a compact Hausdorff space and \( H \) be a Hilbert space. Then \( E = C(X, H) \), the space of all continuous \( H \)-valued functions on \( X \), is a Banach space and it is a left Banach \( C(X) \)-module with the module action defined by \( \pi_l(f, \Lambda) (x) = f(x) \Lambda(x) \) for all \( f \in C(X), \Lambda \in E, x \in X \). Also we define a \( C(X) \)-valued inner product \( \langle \ldots \rangle \) on \( E \) by \( \langle \Lambda, \Gamma \rangle (x) = \langle \Lambda(x), \Gamma(x) \rangle \) for all \( \Lambda, \Gamma \in E, x \in X \). It is easy to verify that \( E \) is a left \( C(X) \)-Hilbert module.

Now let \( h \) be an arbitrary element of Hilbert space \( H \) with \( ||h|| = 1 \). The map \( \Lambda_0 : X \rightarrow H \) defined by \( \Lambda_0(x) = h \) for all \( x \in X \) is a continuous \( H \)-valued function on \( X \), therefore we have \( \Lambda_0 \in E \) and it is easy to see that \( \epsilon(A_0, A_0) = I_{C(X)} \). So \( \pi_\Lambda_0 \) is a product on \( E \) that making it into a Banach algebra denoted by \((E, \pi_\Lambda_0)\).

Theorem 2.2. [8] For a bounded bilinear map \( f : X \times Y \rightarrow Z \) the following statements are equivalent:

(i) \( f \) is regular;
(ii) \( f'''''''' = f''''''' \);
(iii) \( f''''''''(Z', X'') \subseteq Y'' \);
(iv) the linear map \( x \mapsto f'(z', x) : X \rightarrow Y' \) is weakly compact for every \( z' \in Z' \).

Theorem 2.3. Let \( E \) be a left Hilbert \( A \)-module and let for all \( x' \in E' \) the bounded linear map \( T_{x'} : A \rightarrow E' \) defined by \( T_{x'}(a) = \pi_l'(x', a) \) be weakly compact. Then the Banach algebra \((E, \pi_e)\) is Arens regular.

Proof. Let \( \varphi : E \rightarrow A \) be defined by \( \varphi(x) = \epsilon(x, e) \), then \( \varphi \) is a bounded linear map and let \( \pi_l : A \times E \rightarrow E \) be the left module action of \( A \) on \( E \), thus \( \pi_e(x, y) = \pi_l(\varphi(x), y) \). Now suppose that \( x, y \in E, x' \in \ldots \)
\[ E', x'' \in E'' \]. So we have:
\[
< \pi'_e(x', y) > = < x', \pi_e(x, y) > = < x', \pi_i(\varphi(x), y) > = < \pi_i'(x', \varphi(x)), y > .
\]
\[
< \pi''_e(x'', x'), x > = < x'', \pi'_e(x', x) > = < x'', \pi_i'(x', \varphi(x)) > = < \pi_i''(x'', x'), \varphi(x) > = < \varphi(\pi_i''(x'', x')), x > .
\]
\[
< \pi'''_e(x'', y''), x' > = < \pi''_e(y'', x''), x' > = < \pi''_e(y'', \varphi(x''), x') > = < \varphi(\pi''_e(y'', \varphi(x''), x')), x > .
\]

Therefore \( \pi'''_e(x'', y'') = \pi''_e(\varphi(x''), y'') \) (1). Now
\[
< \pi'_e(x', y) > = < x', \pi_e(y, x) > = < x', \pi_i(y, x) > = < \pi_i'(x', y) > = < \varphi^*(x''), \pi''_e(y'', x') > = < \pi_i''(\varphi^*(x''), y''), x' > .
\]
\[
< \pi'''_e(x'', y''), x' > = < \pi'''_e(y'', x''), x' > = < \pi'''_e(y'', \varphi^*(x''), x') > = < \varphi^*(\pi'''_e(y'', \varphi^*(x''), x')), x > .
\]

So we have \( \pi'''_e(x'', y'') = \pi'''_e(\varphi^*(x''), y'') \) (2).

Now, since for all \( x' \in E' \) the bounded linear mapping \( a \rightarrow \pi_i'(x', a) \) from \( A \) to \( E' \) is weakly compact, so applying Theorem (2.2) for \( \pi_i \) shows that \( \pi_i \) is regular, and finally by (1), (2) we have \( \pi'''_e(x'', y'') = \pi'''_e(\varphi^*(x''), y'') \) for all \( x'', y' \in E'' \), thus \( (E, \pi_e) \) is Arens regular.

**Example 2.4.** Let \( Y \) be a Banach space and \( X \) be a compact Hausdorff space. Then \( C(X, Y) \), the space of all continuous \( Y \)-valued functions on \( X \), is a Banach space and \( M(X, Y) \), the Banach space of all countably additive \( Y \)-valued measures with regular finite variation defined on the Borel \( \sigma \)-algebra \( B_X \) of \( X \), is the topological dual of \( C(X, Y) \) [3].
In particular when $H$ is a Hilbert space $\mathcal{M}(X, H)$ is the topological dual of $C(X, H)$. It is proved that if $Y^*$ is weakly sequentially complete then $\mathcal{M}(X, Y^*)$ is weakly sequentially complete [12]. Now since the Hilbert spaces are reflexive, so the topological dual of $C(X, H)$ is weakly sequentially complete, therefore by [1, Theorem 4.2] we have for all $x' \in E'$ the bounded linear mapping $a \mapsto \pi'_i(a')$ from $A$ to $E'$ is weakly compact. Thus applying the above Theorem shows that $(C(X, H), \pi_{\Lambda_0})$ is an Arens regular Banach algebra.

**Definition 2.5.** Let $E$ be a left Hilbert $A$-module and $e$ be an arbitrary element in $E$ with $||e|| = 1$. We define the set $A_e := \{ _e\langle x, e \rangle : x \in E \}$.

It is easy to verify that $A_e$ is a left ideal in $A$, but it is not closed in general. Indeed, let $A = \{ f : [0, 1] \rightarrow \mathbb{C} : f \text{ is continuous} , f(1) = 0 \}$. Then, $f : [0, 1] \rightarrow \mathbb{C}$ defined by $f(x) = x - 1$ is an element of $A$ and $A_f = \{ A\langle g, f \rangle : g \in A \} = \{ gf^* : g \in A \}$ is not closed, because $f \in \overline{A_f}$ and $f \notin A_f$.

Now we give some conditions under which $A_e$ is a closed ideal in $A$. For instance if $e$ be a element of $E$ such that $_e\langle e, e \rangle = 1_A$ then $A_e = A$, because for all $a \in A$ we have $a = a1_A = a _e\langle e, e \rangle = _e\langle a.e, e \rangle$.

The following definition of a Hilbert bimodule is originally due to Brown, Mingo and Shen [4].

**Definition 2.6.** Let $E$ be an $A$-bimodule. $E$ is said to be a Hilbert $A$-bimodule, when $E$ is a left and right Hilbert $A$-module and satisfies the relation $_e\langle x, y \rangle.z = x. _e\langle y, z \rangle$.

**Proposition 2.7.** Let $A$ be unital and $E$ be a Hilbert $A$-bimodule. If $e$ be an element of $E$ such that $ _e\langle e, e \rangle \in Inv(A)$ then $A_e$ is closed.

**Proof.** Let $b \in \overline{A_e}$, then there exists a sequence $(x_n)_{n \in \mathbb{N}} \subseteq E$ such that $ _e\langle x_n, e \rangle$ convergence to $b$. Thus the sequence $( _e\langle x_n, e \rangle)_{n \in \mathbb{N}} \subseteq A$ is Cauchy. Now we have:

$$||x_n - x_m|| = ||(x_n - x_m) _e\langle e, e \rangle _e\langle e, e \rangle^{-1}||$$

$$\leq ||x_n. _e\langle e, e \rangle - x_m. _e\langle e, e \rangle|| || _e\langle e, e \rangle^{-1}||$$

$$= || _e\langle x_n, e \rangle.e - _e\langle x_m, e \rangle.e|| || _e\langle e, e \rangle^{-1}||$$

$$\leq || _e\langle x_n, e \rangle - _e\langle x_m, e \rangle|| || _e\langle e, e \rangle^{-1}||.$$  

So the sequence $(x_n)_{n \in \mathbb{N}} \subseteq E$ is Cauchy and by the completeness of $E$ there exists an element $x \in E$ such that $x_n$ convergence to $x$. Now by continuity of $A$-valued inner product we conclude that $ _e\langle x_n, e \rangle$ convergence to $ _e\langle x, e \rangle$. Thus $b = _e\langle x, e \rangle$ and $A_e$ is closed. \qed

The following useful Proposition is well-known and its proof is straightforward.
Proposition 2.8. Let $X$ and $Y$ be Banach algebras and $T$ be a continuous homomorphism from $X$ onto $Y$. If $X$ is Arens regular then $Y$ is.

Theorem 2.9. Let $A$ be unital and $E$ be a Hilbert $A$-bimodule, $||e|| = 1$ and $\langle e, e \rangle_E \in \text{Inv}(A)$. Then the Banach algebra $(E, \pi_e)$ is Arens regular.

Proof. In Proposition (2.7) we saw that under the above conditions $A_e$ is a closed ideal in $A$. Now since $A$ is Arens regular so $A_e$ is. We define the map $T : A_e \rightarrow (E, \pi_e)$ by $T(e(x, e)) = x$ for all $x \in E$. $T$ is well-defined because if $e(x, e) = e(y, e)$ we have:

\[
x - y = (x - y)(e(x, e)e^{-1}_E)
= ((x - y)(e(x, e)))(e(x, e)e^{-1}_E)
= (e(x, e)(x - y)(e(x, e)).e)(e(x, e)e^{-1}_E).
\]

And $T$ is continuous because

\[
||x_n - x|| = ||(x_n - x)(e(x, e)e^{-1}_E)||
\leq ||(x_n - x)(e(x, e)e^{-1}_E)||||e(x, e)e^{-1}_E||
= ||e(x_n - x, e)||||e(x, e)e^{-1}_E||
\leq ||e(x_n - x, e)||||e(x, e)e^{-1}_E||.
\]

It is easy to see that $T$ is linear. So it is enough that we show that $T$ is multiplicative

\[
T(e(x, e)e(y, e)) = T(e(x, e)(y, e)) = e(x, e)y
= \pi_e(x, y)
= \pi_e(T(e(x, e)), T(e(y, e))).
\]

By Proposition (2.8) since $T$ is onto, the Banach algebra $(E, \pi_e)$ is Arens regular.

3. Derivations of $(E, \pi_e)$

Let $E$ be a left Hilbert $A$-module, and let $e$ be an element in $E$ with $||e|| = 1$ and $(E, \pi_e)$ be the Banach algebra introduced in previous section.

Lemma 3.1. Let $E$ be a full Hilbert $A$-module and let $a \in A$. Then $a = 0$ if and only if $x.a = 0$ for all $x \in E$ [9].

Theorem 3.2. Let $A$ be unital and $E$ be a left Hilbert $A$-module and let $D : A \rightarrow A$ and $\delta : (E, \pi_e) \rightarrow (E, \pi_e)$ be derivations of Banach algebras such that $\delta(a.x) = D(a).x + a.\delta(x)$. Suppose that $\delta$ is inner implemented by $y$, then
(i) if $E$ is full then $D$ is inner.
(ii) if $A$ is unital and there exists $z \in E$ such that $\langle _{E}z,y \rangle \in \text{Inv}(A)$, then $D$ is inner.

Proof. Let $a$ be an arbitrary element of $A$. Then for all $x \in E$, $\delta(ax) = D(a).x + a.\delta(x)$. So for all $x \in E$

\[
D(a).x = \delta(ax) - a.\delta(x) = \pi_{c}(y,ax) - \pi_{c}(ax,y) - ax(\pi_{c}(y,x) - \pi_{c}(x,y)) = \langle _{E}y,e \rangle .(a.x) - \langle _{E}a.x,e \rangle .y - a.\langle _{E}y,e \rangle .x - \langle _{E}x,e \rangle .y = \langle _{E}y,e \rangle .a.x - \langle _{E}a.x,e \rangle .y - a.\langle _{E}y,e \rangle .x + a.\langle _{E}x,e \rangle .y = \langle _{E}y,e \rangle .a.x - a.\langle _{E}y,e \rangle .x.\]

Hence $D(a).x = (\langle _{E}y,e \rangle .a - a.\langle _{E}y,e \rangle )x$ for all $x \in E$.

(i) Since for all $x \in E$ we have $(D(a) - (\langle _{E}y,e \rangle .a - a.\langle _{E}y,e \rangle ))x = 0$ and $E$ is full, applying Lemma (3.1) for left Hilbert modules shows that $D(a) = \langle _{E}y,e \rangle .a - a.\langle _{E}y,e \rangle$ and $D$ is an inner derivation implemented by $\langle _{E}y,e \rangle$.

(ii)Since for all $x \in E$ in particular for $z$, $D(a).x = \langle _{E}y,e \rangle .a.x - a.\langle _{E}y,e \rangle .x$, we conclude that $\langle _{E}D(a).z,y \rangle = \langle _{E}(\langle _{E}y,e \rangle .a - a.\langle _{E}y,e \rangle ).z,y \rangle$ and so $D(a).\langle _{E}z,y \rangle = (\langle _{E}y,e \rangle .a - a.\langle _{E}y,e \rangle )\langle _{E}z,y \rangle$. Now since $\langle _{E}z,y \rangle \in \text{Inv}(A)$ we obtain that $D(a) = \langle _{E}y,e \rangle .a - a.\langle _{E}y,e \rangle$. Thus $D$ is an inner derivation implemented by $\langle _{E}y,e \rangle$.

\[\square\]

Theorem 3.3. Let $E$ be a Hilbert $A$-bimodule, $\langle e,e \rangle _{E} \in \text{Inv}(A)$ and all derivations of $A_{e}$ be inner, then every derivation of $(E,\pi_{e})$ is inner.

Proof. Let $\delta$ be an arbitrary derivation of $(E,\pi_{e})$. We define the mapping $D$ on $A_{e}$ by $D(\langle _{E}x,e \rangle ) = \langle _{E}\delta(x),e \rangle$ for all $x \in E$. It is easy to verify that $D$ is linear, also for all $x,y \in E$ we have:

\[
D(\langle _{E}x,e \rangle \langle _{E}y,e \rangle ) = D(\langle _{E}(\langle _{E}x,e \rangle ,y,e) \rangle ) = \langle _{E}\delta(\langle _{E}x,e \rangle ,y,e) \rangle = \langle _{E}\delta(\pi_{e}(x,y)),e \rangle = \langle _{E}\pi_{e}(\delta(x),y) + \pi_{e}(x,\delta(y)),e \rangle = \langle _{E}\delta(x),y,e \rangle + \langle _{E}(\delta(x),e),y,e \rangle = D(\langle _{E}x,e \rangle )\langle _{E}y,e \rangle + \langle _{E}x,e \rangle D(\langle _{E}y,e \rangle ).\]

So $D$ is a derivation of $A_{e}$ and since every derivation $D : A_{e} \rightarrow A_{e}$ is inner, there exists $t \in E$ such that $D(\langle _{E}x,e \rangle ) = \langle _{E}(\delta(x),e) - \langle _{E}(\pi_{e}(t,x) - \pi_{e}(x,t),e) \rangle .e = 0$. Now since $E$ is a Hilbert bimodule
we have \( \delta(x) - (\pi_e(t, x) - \pi_e(x, t)) \). \( \langle e, e \rangle_E = 0 \) and by invertibility of \( \langle e, e \rangle_E \) we conclude that \( \delta(x) = \pi_e(t, x) - \pi_e(x, t) \) and \( \delta \) is inner. \( \square \)

If in the above theorem we add the conditions under which \( A = A_e \), for example \( \langle e, e \rangle = 1_A \), then we obtain relationship between \( A \) and \( E \).

Now suppose that \( X \) is a compact Hausdorff space and \( H \) is a Hilbert space. For \( E = C(X, H) \) and \( \Lambda_0 \) in Example (2.1) we have \( \langle \Lambda_0, \Lambda_0 \rangle = 1_{C(X)} \), so for every \( f \in C(X) \) we have \( f = f \langle \Lambda_0, \Lambda_0 \rangle = \langle f, \Lambda_0, \Lambda_0 \rangle \). Thus \( C(X) = \{ \langle \Lambda, \Lambda_0 \rangle : \Lambda \in E \} \). Also we notice that \( \Lambda_0 \) is a left unit for Banach algebra \( (E, \pi_{\Lambda_0}) \). So we have:

**Theorem 3.4.** Every derivation of \( (C(X, H), \pi_{\Lambda_0}) \) is zero if and only if \( \Lambda_0 \) is unit element of \( (C(X, H), \pi_{\Lambda_0}) \).

**Proof.** Let \( d \) be an arbitrary derivation of Banach algebra \( (E, \pi_{\Lambda_0}) = (C(X, H), \pi_{\Lambda_0}) \). We define the mapping \( D \) on \( C(X) \) by \( D_{\langle \Lambda, \Lambda_0 \rangle} = \langle \delta(\Lambda), \Lambda_0 \rangle \) for all \( \Lambda \in E \). With the same proof of the above Theorem we have \( D \) is a derivation of \( C(X) \). Now since \( C(X) \) is a commutative \( C^* \)-algebra, \( D \) is zero [5] and so \( D_{\langle \Lambda, \Lambda_0 \rangle} = 0 \) for all \( \Lambda \in E \). Now since \( \Lambda_0 \) is unit element of \( E \) for all \( \Lambda \in E \) we have \( d(\Lambda) = \pi_{\Lambda_0}(d(\Lambda), \Lambda_0) = \langle d(\Lambda), \Lambda_0 \rangle \cdot \Lambda_0 = D_{\langle \Lambda, \Lambda_0 \rangle} \cdot \Lambda_0 = 0 \) and so \( d \equiv 0 \).

For the converse, consider the inner derivation \( d_{\Lambda_0} \) on \( E \) defined by \( d_{\Lambda_0}(\Lambda) = \pi_{\Lambda_0}(\Lambda, \Lambda) - \pi_{\Lambda_0}(\Lambda, \Lambda) \) for all \( \Lambda \in E \). Since every derivation of \( (E, \pi_{\Lambda_0}) \) is zero thus \( d_{\Lambda_0} = 0 \). So for all \( \Lambda \in E \) we have \( \pi_{\Lambda_0}(\Lambda, \Lambda) = \pi_{\Lambda_0}(\Lambda, \Lambda) \) and it shows that \( \pi_{\Lambda_0}(\Lambda, \Lambda) = \Lambda \) and so \( \Lambda_0 \) is unit element of \( (E, \pi_{\Lambda_0}) \). \( \square \)

**References**


A. Sahleh  
Department of Mathematics, University of Guilan, P.O.Box 1914, Rasht, Iran.  
Email: sahlehj@guilan.ac.ir

L. Najarpisheh  
Department of Mathematics, University of Guilan, P.O.Box 1914, Rasht, Iran.  
Email: Najarpisheh@phd.guilan.ac.ir
منظم آرنز بودن و مشتق روي مدول هاي هيبرت با یک ضرب مشخص

عباس سهله* و لیلا نجارپیشه
دانشکده علوم رياضي، دانشگاه گيلان، رشت، ايران

چکیده
فرض كنید E - مدول هيبرت C - جبر و A یک A - جبر و E یک E - مدول هيبرت چپ باشد. در این مقاله ضربی را تعريف مي كنيم كه آن را به یک جبر پاتخ تيديل مي كنیم و نشان خواهيم داد كه تحت شرایط مشخص، E - مرتب همچنين رابطه بين مشتق ها روی E و A باعث مي كنند.