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ON LIE IDEALS AND SYMMETRIC BI-SEMIDERIVATIONS IN PRIME RINGS

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Abstract. In this paper, we investigate the relationship between symmetric bi-semiderivations and Lie ideals of a prime ring. Additionally, we extend some well-known results concerning symmetric biderivations of prime rings to symmetric bi-semiderivations.

1. Introduction

Throughout this paper, R will represent an associative ring and Z will be its center. The symbol $[r,s]$ stands for $rs-sr$ and the symbol $r\circ s$ represent for $rs+sr$. Recall that if $rRs=(0)$ implies $r=0$ or $s=0$, then a ring R is called prime. R is called semiprime if $rRr=(0)$ implies $r=0$. An additive subgroup L of R is said to be a Lie ideal if $[L,R]\subseteq L$. A Lie ideal L is said to be square closed if for all $l\in L$, $l^2\in L$.

An additive map $d:R\to R$ is called derivation if $d(rs)=d(r)s+rd(s)$ holds for all $r,s\in R$. A mapping f is said to be commuting on R if $[f(r),r]=0$ for all $r\in R$. The concept of commuting maps in prime rings with derivations was initiated by Posner [5]. Since then, a lot of work has been done in this concept. The notion of symmetric bi-derivation was introduced by Maksa [4]. A mapping $D:R\times R\to R$ is said to be symmetric if for all $r,s\in R$, $D(r,s)=D(s,r)$. A map $d:R\to R$ defined by $d(r)=D(r,r)$ is called the trace of D, where $D:R\times R\to R$ is a symmetric mapping. It is obvious that if D is bi-additive (i.e., additive in both arguments), then the trace of D

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satisfies the identity $d(r+s) = d(r) + d(s) + 2D(r,s)$ for all $r, s \in R$. Also, we will use the fact that the trace of a symmetric bi-additive mapping is an even function. $D : R \times R \to R$ is called a symmetric bi-derivation if $D(rs,t) = D(r,t)s + rD(s,t)$ is fulfilled for all $r, s, t \in R$. In [7], introduced some results of symmetric bi-derivations on prime and semiprime rings, then the similar results on Lie ideals of R obtained in ([2], [8]). In [1], Ali and Kumar investigated cases where a nonzero square closed $*$-Lie ideal U of a $*$-prime ring R of $charR \neq 2^n - 2$ is central. Rehman and Ansari investigated the commutativity of prime and semiprime rings with symmetric bi-derivations in [6].

The notion of symmetric bi-semiderivations on prime rings is described in [9]. A symmetric bi-additive function $D : R \times R \to R$ is called a symmetric bi-semiderivation associated with a function $f : R \to R$ (or simply a symmetric bi-semiderivation) if $D(rs,t) = D(r,t)f(s) + rD(s,t) = D(r,t)s + f(r)D(s,t)$ and $d(f(r)) = f(d(r))$ for all $r, s, t \in R$, where $d : R \to R$ is the trace of D. Let R be a commutative ring and $B := \left\{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} : \ a, b \in R \right\}$. Then B is a ring with matrix addition and multiplication.

$D : B \times B \to B$ by $\left(\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} c & d \\ 0 & 0 \end{pmatrix} \right) \mapsto \begin{pmatrix} 0 & bd \\ 0 & 0 \end{pmatrix}$ is a symmetric bi-semiderivation, where $f : B \to B$ defined by $\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \mapsto \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}$.

The aim of this paper is to obtain some results symmetric bi-semiderivations on prime rings.

2. Preliminaries

Lemma 2.1. *([5], Lemma 1) Let R be a prime ring, d be a derivation of R and $r \in R$. For all $s \in R$, if $rd(s) = 0$ then $d = 0$ or $r = 0$.*

Lemma 2.2. *([3], Lemma 4) Provided that $L \nsubseteq Z$ is a Lie ideal of a prime ring R with $charR \neq 2$ and $r, s \in R$ such that $rLs = (0)$, then $r = 0$ or $s = 0$.*

Lemma 2.3. *Suppose that R is a prime ring with $charR \neq 2$ and A is a non-zero left (or right) ideal of R. Let D be a symmetric bi-semiderivation and d be the trace of D. If $d(a) = 0$ for all $a \in A$, then $d = 0$, that is, $D = 0$.*

Proof. The proof is the similar with proof of ([8], Lemma 4). □

Lemma 2.4. *([9], Lemma 4) Suppose that R is a prime ring with $charR \neq 2$. Let D be a symmetric bi-semiderivation of R, d be the* *trace of D and a be an element of R. If $[a, d(r)] = 0$ for all $r \in R$, then $a \in Z$ or $d = 0$.*

Theorem 2.5. *([9], Theorem 1) Let $D \neq 0$ be a symmetric bi-semiderivations of a prime ring R associated with a function f (not necessarily surjective). Then f is a homomorphism of R.*

Theorem 2.6. *([7], Theorem 4) Let R be a 2-torsion free semiprime ring, $D : R\times R \to R$ be a symmetric biderivation such that $D(d(r), r) = 0$ for all $r \in R$, where d is the trace of D. Then, $D = 0$.*

Theorem 2.7. *Suppose that R is a noncommutative prime ring with $char R \neq 2$ and A is a non-zero ideal of R. Let D be a symmetric bi-semiderivation associated with a surjective function f such that $D(A, A) \subseteq A$ and d be the trace of D. If d is commuting on A, then $D = 0$.*

Proof. We have

$$[d(a), a] = 0 \text{ for all } a \in A. \tag{2.1}$$

The linearization of (2.1) gives

$$[d(a), b] + [d(b), a] + 2[D(a, b), a] + 2[D(a, b), b] = 0 \text{ for all } a, b \in A. \tag{2.2}$$

Substituting $-a$ for a in (2.2), we have

$$[d(a), b] - [d(b), a] + 2[D(a, b), a] - 2[D(a, b), b] = 0. \tag{2.3}$$

From (2.2) and (2.3), using $char R \neq 2$ we arrive

$$[d(a), b] + 2[D(a, b), a] = 0. \tag{2.4}$$

Now, we write ab instead of b in (2.4). Thus,

$$\begin{aligned} 0 &= [d(a), ab] + 2[d(a)f(b) + aD(a, b), a] \\ &= a[d(a), b] + 2d(a)[f(b), a] + 2a[D(a, b), a] \end{aligned}$$

which implies

$$d(a)[a, f(b)] = 0 \tag{2.5}$$

according to (2.4). Since f is a surjective function, we have

$$d(a)[a, c] = 0, \text{for all } a \in A, c \in R. \tag{2.6}$$

Hence, for any $a \notin Z$, $d(a) = 0$ from (2.6) and Lemma 2.1. (note that for any fixed $a \in R$, a mapping $b \mapsto [a, b]$ is a derivation) Let $a \in Z$, $c \notin Z$. Then $-c \notin Z$ and $a+c \notin Z$. Thus, $0 = d(a+c) = d(a)+2D(a, c)$ and $0 = d(a) - 2D(a, c)$. Therefore, $d(a) = 0$ for all $a \in A$ and $D = 0$ by Lemma 2.3. ∎

We will use the well known following commutator identities for a ring R:

(*i*) $[r_1r_2, r_3] = r_1[r_2, r_3] + [r_1, r_3]r_2$,

(*ii*) $[r_1, r_2r_3] = r_2[r_1, r_3] + [r_1, r_2]r_3$,

(*iii*) $r_1 \circ r_2r_3 = (r_1 \circ r_2)r_3 - r_2[r_1, r_3] = r_2(r_1 \circ r_3) + [r_1, r_2]r_3$,

(*iv*) $(r_1r_2) \circ r_3 = r_1(r_2 \circ r_3) - [r_1, r_3]r_2 = (r_1 \circ r_3)r_2 + r_1[r_2, r_3]$.

Remark 2.8. Let R be a prime ring and L be a non-zero square closed Lie ideal of R. For all $l, m \in L$, we have $lm + ml = (l + m)^2 - l^2 - m^2$ and $l^2 \in L$ imply that $lm + ml \in L$. Then we get $2lm \in L$ for all $l, m \in L$. So, we obtain $2r[l, m] = 2[l, rm] - 2[l, r]m \in L$ and $2[l, m]r = 2[l, mr] - 2m[l, r] \in L$ for all $r \in R$. This provides that $2R[L, L] \subseteq L$ and $2[L, L]R \subseteq L$.

3. Lie ideals and symmetric bi-semiderivations

Example 3.1. Let R be a commutative ring and $F := \left\{ \begin{pmatrix} 0 & r & s \\ 0 & 0 & 0 \\ 0 & 0 & s \end{pmatrix} \mid r, s \in R \right\}$. Then F is a ring with matrix addition and multiplication. $D : F \times F \to F$ defined by $\left(\begin{pmatrix} 0 & r & s \\ 0 & 0 & 0 \\ 0 & 0 & s \end{pmatrix}, \begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & 0 & y \end{pmatrix} \right) \mapsto \begin{pmatrix} 0 & 0 & sy \\ 0 & 0 & 0 \\ 0 & 0 & sy \end{pmatrix}$ is a symmetric bi-semiderivation, where $f : F \to F$ defined by $\begin{pmatrix} 0 & r & s \\ 0 & 0 & 0 \\ 0 & 0 & s \end{pmatrix} \mapsto \begin{pmatrix} 0 & r & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

In [6], authors showed the following properties:

Let R be a prime ring with $charR \neq 2$, L be a square closed Lie ideal of R and D be a symmetric biderivation with the trace d.

(*i*) If $[d(l), m] \in Z$ for all $l, m \in L$, then $d = 0$ or $L \subseteq Z$.

(*ii*) If $[d(l), l] = 0$ for all $l \in L$, then $d = 0$ or $L \subseteq Z$.

(*iii*) If $d([l, m]) - [d(l), m] \in Z$ for all $l, m \in L$, then $d = 0$ or $L \subseteq Z$.

(*iv*) If $d(l \circ m) - [d(l), m] \in Z$ for all $l, m \in L$, then $d = 0$ or $L \subseteq Z$.

(*iv*) If $d(l) \circ m - [d(l), m] \in Z$ for all $l, m \in L$, then $d = 0$ or $L \subseteq Z$.

(*v*) If $d(l) \circ d(m) - [d(l), m] \in Z$ for all $l, m \in L$, then $d = 0$ or $L \subseteq Z$.

(*vi*) If $d(lm) - d(l)m - ld(m) \in Z$ holds for all $l, m \in L$, then either $d = 0$ or $L \subseteq Z$.

(*vii*) If $d([l, m]) - [d(l), m] - [l, d(m)] \in Z$ holds for all $l, m \in L$, then either $d = 0$ or $L \subseteq Z$.

Same expressions are provided for symmetric bi-semiderivation D. The proofs are similar, so we omit them.

Theorem 3.2. *Assume that R is a prime ring with $charR \neq 2$ and L is a non-zero Lie ideal of R. Let D be a symmetric bi-semiderivation associated with a surjective function f and d be the trace of D.*

(i) If $d(L) = 0$, then $d = 0$ or $L \subseteq Z$.
(ii) If $d(L) \subseteq Z$ and L is a square closed Lie ideal, then $d = 0$ or $L \subseteq Z$.

Proof. (i) We suppose that

$$d(l) = 0 \text{ for all } l \in L. \tag{3.1}$$

Since $charR \neq 2$, for all $l, m \in L$, the linearization of (3.1) gives

$$D(l, m) = 0. \tag{3.2}$$

Putting $[l, r]$ instead of l in (3.2), where $r \in R$, we have

$$D(l, m)f(r) + lD(r, m) - D(r, m)l - f(r)D(l, m) = 0$$

which implies that

$$[l, D(r, m)] = 0. \tag{3.3}$$

Replace in (3.3) r by rn, where $n \in L$. Then

$$D(r, m)[l, n] = 0 \text{ for all } l, m, n \in L,\ r \in R. \tag{3.4}$$

Substituting rs for r in (3.4), where $s \in R$. We obtain

$$D(r, m)s[l, n] = 0 \text{ for all } l, m, n \in L,\ r, s \in R.$$

By primeness of R, $D(r, m) = 0$ for all $r \in R$, $m \in L$ or $[l, n] = 0$ for all $l, n \in L$. If $[l, n] = 0$ for all $l, n \in L$, then $L \subseteq Z$. We suppose that

$$D(r, m) = 0 \text{ for all } r \in R, m \in L. \tag{3.5}$$

Let m be $[m, r]$ in (3.5). Then, we get

$$[m, d(r)] = 0 \text{ for all } m \in L,\ r \in R. \tag{3.6}$$

By (3.6) and using Lemma 2.4, we have that $d = 0$ or $L \subseteq Z$.
(ii) We suppose that

$$d(l) \in Z \text{ for all } l \in L. \tag{3.7}$$

The linearization of (3.7) gives

$$D(l, m) \in Z \text{ for all } l, m \in L, \tag{3.8}$$

since we have assumed that $charR \neq 2$. Replace l by l^2 in (3.8), we have

$$lD(l, m) + D(l, m)f(l) \in Z \text{ for all } l, m \in L.$$

In particular, $ld(l)+d(l)r \in Z$ for all $l \in L, r \in R$, since f is surjective. Commuting with l and using $d(l) \in Z$, we obtain

$$d(l)[r,l] = 0 \text{ for all } l \in L, r \in R.$$

Then we get $d(l) = 0$ or $[r,l] = 0$ for all $l \in L, r \in R$. Therefore, in two cases, we arrive at $d = 0$ or $L \subseteq Z$, from Theorem (i). □

Lemma 3.3. *Assume that R is a prime ring with $charR \neq 2$, L is a nonzero Lie ideal of R and f is a surjective homomorphism on R. For all $l, m \in L$, if $f([l,m]) = 0$ then either $f(L) = 0$ or $f(L) \subseteq Z$.*

Proof. We have

$$f([l,m]) = 0 \text{ for all } l, m \in L. \tag{3.9}$$

Replacing l by $[r,l]$, $r \in R$ and using f is a homomorphism, we get

$$0 = f([r,m])f(l) - f(l)f([r,m]). \tag{3.10}$$

Taking r by rs, $s \in R$ in (3.10), we have

$$f([r,m])f(s)f(l) + f(r)f([s,m])f(l) - f(l)f([r,m])f(s) - f(l)f(r)f([s,m]) = 0$$

for all $l, m \in L$, $r, s \in R$. From (3.10), we obtain

$$f([r,m])[f(s), f(l)] + [f(r), f(l)]f([s,m]) = 0.$$

If we write $s = m$, then we arrive

$$f([r,m])[f(m), f(l)] = 0 \text{ for all } l, m \in L,\ r \in R.$$

In the last relation, replace r by rt, $t \in R$, we get $f([r,m])f(t)[f(m), f(l)] = 0$. Since R is prime ring, f is surjective, we have $f([r,m]) = 0$ or $[f(m), f(l)] = 0$ for all $l, m \in L$, $r \in R$. This implies that $f(L) = 0$ or $f(L) \subseteq Z$. The proof of Lemma is completed. □

Lemma 3.4. *Assume that R is a prime ring with $charR \neq 2$, L is a nonzero Lie ideal of R and D is a symmetric bi-semiderivation associated with a surjective function f and d is the trace of D. If $d(L) = 0$, then $f(L) = 0$ or $f(L) \subseteq Z$ or $D = 0$.*

Proof. Given that

$$d(l) = 0 \text{ for all } l \in L. \tag{3.11}$$

Linearizing (3.11) and using $charR \neq 2$, we have

$$D(l,m) = 0 \text{ for all } l, m \in L. \tag{3.12}$$

Let us replace l by $[l,r]$, $r \in R$ in (3.12), we obtain

$$lD(r,m) - D(r,m)f(l) = 0 \text{ for all } l, m \in L \text{ and } r \in R. \tag{3.13}$$

Replacing r by rn_1, $n_1 \in L$ and using (3.12), we have

$$lD(r,m)f(n_1) - D(r,m)f(n_1 l) = 0. \quad (3.14)$$

If we multiply (3.13) by $f(n_1)$ to the right, we get

$$lD(r,m)f(n_1) - D(r,m)f(l\ n_1) = 0 \quad (3.15)$$

From (3.14) and (3.15), we find that

$$D(r,m)f([l,n_1]) = 0 \text{ for all } l,m,n_1 \in L,\ r \in R. \quad (3.16)$$

In (3.16), we replace r by rn_2, $n_2 \in L$

$$D(r,m)f(n_2)f([l,n_1]) = 0.$$

Since f is surjective, $D(r,m)Rf([l,n_1]) = (0)$ for all $l,m,n_1 \in L$ and $r \in R$. By primeness of R, either $D(r,m) = 0$ or $f([l,n_1]) = 0$. If $D(r,m) = 0$ for all $r \in R$, $m \in L$, then replacing m by $[m,s]$, $s \in R$,

$$f(m)D(r,s) - D(r,s)f(m) = 0 \text{ for all } m \in L,\ r,s \in R. \quad (3.17)$$

Putting s by sn_3, $n_3 \in L$ in (3.17), we have

$$f(m)D(r,s)f(n_3) - D(r,s)f(n_3)f(m) = 0. \quad (3.18)$$

Multiplying (3.17) from right $f(n_3)$ and subtract (3.18), we get

$$D(r,s)f([m,n_3]) = 0 \text{ for all } r,s \in R \text{ and } m,n_3 \in L.$$

In the last equation, if we replace r by tr, $t \in R$, we have
$D(t,s)f(r)f([m,n_3]) = 0$ for all $r,s,t \in R$ and $m,n_3 \in L$. Since R is prime and f is surjective $D = 0$ or $f([m,n_3]) = 0$ for all $m,n_3 \in L$. Hence, Lemma 3.3 gives the proof of Lemma. □

Theorem 3.5. *Let R be a prime ring with $charR \neq 2$, L be a nonzero square closed Lie ideal of R and D be a symmetric bi-semiderivation associated with surjective function f. If $D([l_1,l_2],[m_1,m_2]) = 0$ for all $l_1,l_2,m_1,m_2 \in L$, then $L \subseteq Z$ or $f(L) = 0$ or $f(L) \subseteq Z$ or $D(L,L) = 0$.*

Proof. Suppose that

$$D([l_1,l_2],[m_1,m_2]) = 0 \text{ for all } l_1,l_2,m_1,m_2 \in L. \quad (3.19)$$

Replacing l_2 by $2l_2l_1$ in (3.19) and using (3.19), we get

$$[l_1,l_2]D(l_1,[m_1,m_2]) = 0 \text{ for all } l_1,l_2,m_1,m_2 \in L,$$

since $charR \neq 2$. Replacing m_1 by $2m_1m_2$ and using $charR \neq 2$, we have

$$[l_1,l_2]f([m_1,m_2])D(l_1,m_2) = 0 \text{ for all } l_1,l_2,m_1,m_2 \in L. \quad (3.20)$$

Now, we substituting $[r, l_2]$ for l_2 in (3.20) and we using (3.20), for all $l_1, l_2, m_1, m_2 \in L$, $r \in R$

$$[[l_1, r], l_2]\, f([m_1, m_2])D(l_1, m_2) - [l_1, l_2]rf([m_1, m_2])D(l_1, m_2) = 0. \tag{3.21}$$

Since L is a Lie ideal of R, we get $[l_1, r] \in L$. Then, from (3.21), we obtain that $[l_1, l_2]rf([m_1, m_2])D(l_1, m_2) = 0$ for all $l_1, l_2, m_1, m_2 \in L$, $r \in R$. Then, we have either $L \subseteq Z$ or $f([m_1, m_2])D(l_1, m_2) = 0$ for all $l_1, m_1, m_2 \in L$, by primeness of R. Let $f([m_1, m_2])D(l_1, m_2) = 0$. Replacing $m_1$by $[s, m_1]$, $s \in R$, we get

$$[f([s, m_2], f(m_1)]\, D(l_1, m_2) - f([m_1, m_2])f(s)D(l_1, m_2) = 0, \tag{3.22}$$

for all $l_1, m_1, m_2 \in L, s \in R$. Since $[s, m_2] \in L$, we obtain $f([m_1, m_2])f(s)D(l_1, m_2) = 0$ for all $l_1, m_1, m_2 \in L$, $s \in R$. Since f is surjective and R is prime ring, we have either $f([m_1, m_2]) = 0$ or $D(l_1, m_2) = 0$ for all $l_1, m_1, m_2 \in L$. Therefore, the proof is completed in the light of all the obtained results and using Lemma 3.3. □

Now, we consider Theorem 3.5 with the condition $L \nsubseteq Z$.

Theorem 3.6. *Let R be a prime ring with $charR \neq 2$, L be a nonzero square closed Lie ideal of R. Suppose that D is a symmetric bi-semiderivations such that $D([l_1, l_2], [m_1, m_2]) = 0$ for all $l_1, l_2, m_1, m_2 \in L$. If $L \nsubseteq Z$, then $D(L, L) = 0$.*

Proof. We have $D([l_1, l_2], [m_1, m_2]) = 0$. If we replace l_2 by $2l_2l_1$, then we get

$$\begin{aligned} 0 &= D([l_1, l_2]l_1, [m_1, m_2]) \\ &= [l_1, l_2]D(l_1, [m_1, m_2]). \end{aligned}$$

Taking m_1 by $2m_1m_2$ and using $charR \neq 2$, we get

$$0 = [l_1, l_2][m_1, m_2]D(l_1, m_2) \text{ for all } l_1, l_2, m_1, m_2 \in L. \tag{3.23}$$

Substituting $2l_2l_3$ for l_2 in (3.23) and using $charR \neq 2$, we get

$$[l_1, l_2]l_3[m_1, m_2]D(l_1, m_2) = 0.$$

From Lemma 2.2, we obtain $[l_1, l_2] = 0$ or $[m_1, m_2]D(l_1, m_2) = 0$. Using our hypothesis, we get $[m_1, m_2]D(l_1, m_2) = 0$ for all $l_1, m_1, m_2 \in L$. Replacing m_1 by $2m_3m_1$ in the above relation, $[m_3, m_2]m_1D(l_1, m_2) = 0$ for all $l_1, m_1, m_2, m_3 \in L$. Again using Lemma 2.2, we have $[m_3, m_2] = 0$ or $D(l_1, m_2) = 0$. By our assumption, we arrive that $D(l_1, m_2) = 0$ for all $l_1, m_2, \in L$. □

Example 3.7. Let $R = \left\{ A = \begin{pmatrix} 0 & r \\ 0 & s \end{pmatrix} \mid r, s \in \mathbb{Z} \right\}$, where $\mathbb{Z}$ is the set of all integers. Consider $L = \left\{ \begin{pmatrix} 0 & r \\ 0 & 0 \end{pmatrix} \mid r \in \mathbb{Z} \right\}$. Hence, R is a ring and L is a Lie deal of R. Since $\begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} R \begin{pmatrix} 0 & 3 \\ 0 & 0 \end{pmatrix} = (0)$, we have that R is not prime. We define $D : R \times R \to R$ by $D\left(\begin{pmatrix} 0 & r \\ 0 & s \end{pmatrix}, \begin{pmatrix} 0 & t \\ 0 & w \end{pmatrix} \right) = \begin{pmatrix} 0 & rt \\ 0 & 0 \end{pmatrix}$ and $f : R \to R$ by $f \begin{pmatrix} 0 & r \\ 0 & s \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & s \end{pmatrix}$. Then D is a symmetric bi-semiderivation associated with f. We can see that $D([A, B], [C, D]) = 0$ for all $A, B, C, D \in L$. Since L is noncentral, we arrive that the primeness of R in the above result is not redundant.

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