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STUDY OF THE STRUCTURE OF QUOTIENT RINGS SATISFYING ALGEBRAIC IDENTITIES

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ABSTRACT. Assuming that $\mathcal{R}$ is an associative ring with prime ideal P, this paper investigates the commutativity of the quotient ring $\mathcal{R}/P$, as well as the possible forms of generalized derivations satisfying certain algebraic identities on $\mathcal{R}$. We give results on strong commutativity, preserving generalized derivations of prime rings, using our theorems. Finally, an example is given to show that the restrictions on the ideal P are not superfluous.

1. INTRODUCTION

In all that follows, $\mathcal{R}$ always denotes an associative ring with center $Z(\mathcal{R})$ and $\mathcal{C}$ is the extended centroid of $\mathcal{R}$ (we refer the reader to [11] for more information about these objects). As usual, the symbols $[s,t]$ and $s \circ t$ denote the commutator $st - ts$ and the anticommutator $st + ts$, respectively. Recall that a ring $\mathcal{R}$ is prime if $x\mathcal{R}y = \{0\}$ implies $x = 0$ or $y = 0$, and $\mathcal{R}$ is semiprime if $x\mathcal{R}x = \{0\}$ implies $x = 0$.

The study of commutativity preserving mappings has been an active research area in matrix theory, operator theory, and ring theory (see [13], [17] for references). According to [7], let $\mathcal{S}$ be a subset of $\mathcal{R}$. A map $F : \mathcal{R} \to \mathcal{R}$ is said to be strongly commutativity preserving (SCP) on $\mathcal{S}$ if $[F(x), F(y)] = [x, y]$ for all $x, y \in \mathcal{S}$. In [6], Bell and Daif investigated commutativity in rings admitting a derivation which is SCP on a nonzero right ideal. In particular, they proved that if a semiprime ring $\mathcal{R}$ admits a derivation d satisfying $[d(x), d(y)] = [x, y]$

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for all x, y in a right ideal I of $\mathcal{R}$, then $I \subseteq Z(\mathcal{R})$ (see [12] for more information). In particular, $\mathcal{R}$ is commutative if $I = \mathcal{R}$. Later, Deng and Ashraf [18] proved that if there exists a derivation d of a semiprime ring $\mathcal{R}$ and a map $F : I \to \mathcal{R}$ defined on a non-zero ideal I of $\mathcal{R}$ such that $[F(x), d(y)] = [x, y]$ for all $x, y \in I$, then $\mathcal{R}$ contains a non-zero central ideal. In particular, they showed that $\mathcal{R}$ is commutative if $I = \mathcal{R}$. Recently, this result was extended to Lie ideals and symmetric elements of prime rings by Lin and Liu in [10] and [8], respectively, and to the case of generalized derivations by Ma and Xu in [9]. There is also a growing literature on strong commutativity preserving (SCP) maps and derivations (for references see [6], [13], [9], etc.) In [1], Ali et al. showed that if $\mathcal{R}$ is a semiprime ring and f is an endomorphism which is a strong commutativity preserving (simple, SCP) map on a non-zero ideal U of $\mathcal{R}$, then f commutes on U. In [16], Samman proved that an epimorphism of a semiprime ring is strongly commutativity preserving if and only if it is centralizing. Derivations as well as SCP mappings have been studied extensively by researchers also in the context of operator algebras, prime rings, and semiprime rings. Many related generalizations of these results can be found in the literature (see for example [13], [19], [3], [2], and [4]).

In this paper, we discuss the notion of a generalized derivation that satisfies one of the following conditions:

(**i**) $[F(x), F(y)] + H([x, y]) \in P$, for all $x, y \in \mathcal{R}$,

(**ii**) $F(x) \circ F(y) + H(x \circ y) \in P$, for all $x, y \in \mathcal{R}$,

(**iii**) $[F(x), F(y)] + H(x \circ y) \in P$, for all $x, y \in \mathcal{R}$,

(**iv**) $F(x) \circ F(y) + H([x, y]) \in P$, for all $x, y \in \mathcal{R}$,

where P is a prime ideal of $\mathcal{R}$, F is a generalized derivation, and H is a multiplier of $\mathcal{R}$.

Finally, a concrete example is given to show that the restrictions on our assumptions about the results are not superfluous.

2. Results

In this part, we will talk about some well-known results in ring theory, which will be used a lot in the following sections.

(i) $[x, yz] = y[x, z] + [x, y]z$.

(ii) $[xy, z] = [x, z]y + x[y, z]$.

(iii) $xy \circ z = (x \circ z)y + x[y, z] = x(y \circ z) - [x, z]y$.

(iv) $x \circ yz = y(x \circ z) + [x, y]z = (x \circ y)z + y[z, x]$.

Lemma 2.1. [5, Lemma 2.1] *Let $\mathcal{R}$ be a ring, P be a prime ideal of $\mathcal{R}$, and d a derivation of $\mathcal{R}$. If $[d(x), x] \in P$ for all $x \in \mathcal{R}$, then $d(\mathcal{R}) \subseteq P$ or $\mathcal{R}/P$ is commutative.*

Lemma 2.2. [14, Lemma] *Let $\mathcal{R}$ be a prime ring. If functions $F : \mathcal{R} \longrightarrow \mathcal{R}$ and $G : \mathcal{R} \longrightarrow \mathcal{R}$ are such that $F(x)yG(z) = G(x)yF(z)$ for all $x, y, z \in \mathcal{R}$, and $F \neq 0$, then there exists λ in the extended centroid of $\mathcal{R}$ such that $G(x) = \lambda F(x)$ for all $x \in \mathcal{R}$.*

The following two Lemmas are also used to prove our theorems. The primary goal is to establish a connection between the commutativity of rings $\mathcal{R}/\mathcal{P}$ and the behavior of their generalized derivations. The next lemma is a generalization of [5, Lemma 2.1].

Lemma 2.3. *Let $\mathcal{R}$ be a ring and P be a prime ideal of $\mathcal{R}$. If $\mathcal{R}$ admits a multiplicative generalized derivation F associated with a derivation d, such that $\mathcal{R}$ satisfies one of the following assertions:*

(i) $[x, F(y)] \in P$ *for all* $x, y \in \mathcal{R}$,

(ii) $x \circ F(y) \in P$ *for all* $x, y \in \mathcal{R}$,

then $d(\mathcal{R}) \subseteq P$ or $\mathcal{R}/P$ is commutative.

Proof. (i) Suppose that

$$[x, F(y)] \in P \quad \text{for all } \ x, y \in \mathcal{R}. \tag{2.1}$$

Replacing y by yt in (2.1), we obtain

$$F(y)[x, t] + [x, F(y)]t + y[x, d(t)] + [x, y]d(t) \in P \text{ for all } x, y, t \in \mathcal{R}.$$

Using (2.1), we get

$$F(y)[x, t] + [x, y]d(t) + y[x, d(t)] \in P \text{ for all } x, y, t \in \mathcal{R}. \tag{2.2}$$

For $x = t$ in (2.2), it follows that

$$[t, y]d(t) + y[t, d(t)] \in P \quad \text{for all } \ y, t \in \mathcal{R}. \tag{2.3}$$

Taking ry instead of y in (2.3) and using (2.3), we conclude that

$$[t, r]yd(t) \in P \quad \text{for all } \ r, y, t \in \mathcal{R}.$$

Equivalently,

$$[t, r]Rd(t) \subseteq P \quad \text{for all } \ r, t \in \mathcal{R}.$$

By primness of P, we arrive at

$$[t, r] \in P \quad \text{or } \ d(t) \in P \quad \text{for all } \ r, t \in \mathcal{R}. \tag{2.4}$$

If we set $A = \{t \in \mathcal{R} \mid [t, r] \in P \text{ for all } r \in \mathcal{R}\}$ and $B = \{t \in \mathcal{R} \mid d(t) \in P\}$. Then A and B are two additive subgroups of $\mathcal{R}$ such that $A \cup B = \mathcal{R}$. Since a group cannot be a union of two of its proper subgroups, we must conclude that $\mathcal{R} = A$ or $\mathcal{R} = B$. Therefore, $d(\mathcal{R}) \subseteq P$ or $\mathcal{R}/P$ is commutative.
(ii) Using the same techniques that as used in the proof of (i) with minor modifications, we can easily arrive at our result. □

Lemma 2.4. *Let $\mathcal{R}$ be a ring and P be a prime ideal of $\mathcal{R}$. If $\mathcal{R}$ admits a generalized derivation F associated with a derivation d such that $\mathcal{R}$ satisfies any of the following assertions:*

(i) $[x, F(x)] \in P$ *for all* $x \in \mathcal{R}$,

(ii) $x \circ F(x) \in P$ *for all* $x \in \mathcal{R}$,

then $d(\mathcal{R}) \subseteq P$ *or* $\mathcal{R}/P$ *is commutative.*

Proof. (i) Assuming that

$$[x, F(x)] \in P \quad \text{for all} \ \ x \in \mathcal{R}. \tag{2.5}$$

Linearizing Eq. (2.5), we obtain

$$[x, F(y)] + [y, F(x)] \in P \ \text{ for all } x, y \in \mathcal{R}. \tag{2.6}$$

Replacing y by yx in (2.6), and using it with (2.5) we obtain

$$y[x, d(x)] + [x, y]d(x) \in P \ \text{ for all } x, y \in \mathcal{R}. \tag{2.7}$$

Putting yz instead of y in (2.7), where $z \in \mathcal{R}$ and using it again, we get

$$[x, y]zd(x) \in P \quad \text{for all } x, y, z \in \mathcal{R}.$$

Since P is prime ideal of $\mathcal{R}$, we arrive at

$$[x, y] \in P \ \text{ or } \ d(x) \in P \quad \text{for all } x, y \in \mathcal{R}. \tag{2.8}$$

Suppose that $d(\mathcal{R}) \nsubseteq P$. There exists $x \in \mathcal{R}$ such that $d(x) \notin P$. By (2.8), we get $[x, y] \in P$ for all $y \in \mathcal{R}$ which implies that $\bar{x} \in Z(\mathcal{R}/P)$. Let $z \in \mathcal{R}$ such that $\bar{z} \notin Z(\mathcal{R}/P)$. Then, there exists $y_0 \in \mathcal{R}$ such that $[z, y_0] \notin P$. Therefore, from (2.8), we find that $d(z) \in P$. On the other hand, since $d(x) \notin P$, we can derive $d(x + z) \notin P$. Using (2.8) again, the last expression gives $[x + z, y] \in P$ for all $y \in \mathcal{R}$, which forces that $[z, y] \in P$ for all $y \in \mathcal{R}$; a contradiction.

(ii) Suppose that

$$x \circ F(x) \in P \quad \text{for all} \ \ x \in \mathcal{R}. \tag{2.9}$$

Linearizing (2.9), we get

$$x \circ F(y) + y \circ F(x) \in P \ \text{ for all } x, y \in \mathcal{R}. \tag{2.10}$$

Substituting yx for y in (2.10),and using it again, we find that

$$y(x \circ d(x)) + [x, y]d(x) + y[x, F(x)] \in P \ \text{ for all } x, y \in \mathcal{R}. \tag{2.11}$$

Replacing y by yz in (2.11), where $z \in \mathcal{R}$ and we using it again, we obtain

$$[x, y]zd(x) \in P \quad \text{for all } x, y, z \in \mathcal{R}. \tag{2.12}$$

We continue with the same techniques as used in (i), and we get the required result. $\square$

Corollary 2.5. *Let $\mathcal{R}$ be a prime ring. If $\mathcal{R}$ admits a generalized derivation F associated with a nonzero derivation d, then the following assertions are equivalents :*

(i) $[x, F(x)] = 0$ *for all* $x \in \mathcal{R}$.
(ii) $x \circ F(x) = 0$ *for all* $x \in \mathcal{R}$.
(iii) $\mathcal{R}$ *is a commutative ring.*

Theorem 2.6. *Let $\mathcal{R}$ be a ring and P a prime ideal of $\mathcal{R}$. Suppose that $\mathcal{R}$ admits a multiplier H and a generalized derivation F associated with a nonzero derivation d of $\mathcal{R}$ such that $d(P) \subseteq P$. If $[F(x), F(y)] + H([x, y]) \in P$ for all $x, y \in \mathcal{R}$, then one of the following assertions hold:*

(i) $H(\mathcal{R}) \subseteq P$.
(ii) *There exists $\lambda \in C$ such that $F - \lambda$ maps $\mathcal{R}$ into P with $(\lambda^2 + H)([x, y]) \in P$ for all $x, y \in \mathcal{R}$.*
(iii) $\mathcal{R}/P$ *is a commutative ring.*

Proof. Suppose that $\mathcal{R}/P$ is not a commutative ring and

$$[F(x), F(y)] + H([x, y]) \in P \text{ for all } x, y \in \mathcal{R}. \tag{2.13}$$

Replacing x by xt in (2.13) and using it, we conclude that

$$F(x)[t, F(y)] + x[d(t), F(y)] + [x, F(y)]d(t) + H(x)[t, y] \in P \\ \text{for all } x, y, t \in R. \tag{2.14}$$

Substituting ux for x in (2.14), we find that

$$F(u)x[t, F(y)] + ud(x)[t, F(y)] + ux[d(t), F(y)] + u[x, F(y)]d(t)+ \\ [u, F(y)]xd(t) + uH(x)[t, y] \in P. \tag{2.15}$$

Left-multiplying (2.14) by u and comparing with (2.15), we get

$$(F(u)x - uF(x))[t, F(y)] + ud(x)[t, F(y)] + [u, F(y)]xd(t) \in P \\ \text{for all } x, y, u, t \in \mathcal{R}. \tag{2.16}$$

Taking $t = F(y)$ in (2.16), we obtain $[u, F(y)]\mathcal{R}d(F(y)) \subseteq P$ for all $u, y \in \mathcal{R}$.
By primeness of P, it follows that for each y in $\mathcal{R}$ either $[u, F(y)] \in P$ for all $u \in \mathcal{R}$ or $d(F(y)) \in P$.
Let $A = \{y \in \mathcal{R} \mid [u, F(y)] \in P \text{ for all } u \in \mathcal{R}\}$ and $B = \{y \in \mathcal{R} \mid d(F(y)) \in P\}$. Clearly, A and B are additive subgroups of $\mathcal{R}$ such that $A \cup B = \mathcal{R}$. The fact that a group cannot be a union of two of its proper subgroups, forces us to conclude that either $\mathcal{R} = A$ or $\mathcal{R} = B$.

Assume that $\mathcal{R} = A$. Then by Lemma 2.3(i) and our hypothesis, we get $d(\mathcal{R}) \subseteq P$. In the latter case, from our assumption we get

$$[u, F(yw)] = F(y)[u, w] + [u, F(y)]w + [u, yd(w)] \in P \text{ for all } y, u, w \in \mathcal{R}.$$

Since $[u, F(yw)] \in P$ and $[u, F(y)] \in P$ for all $w, u, y \in P$, it is easy to notice that $F(y)[u, w] \in P$ for all $y, u, w \in \mathcal{R}$. From this, we can easily arrive at $F(y)\mathcal{R}[u, w] \subseteq P$ for all $u, w, y \in \mathcal{R}$. Hence, it follows that $F(\mathcal{R}) \subseteq P$. From our initial hypothesis (2.13), we get

$$H([x, y]) \in P \text{ for all } x, y \in \mathcal{R}. \tag{2.17}$$

In (2.17), replacing x by xt and using it again, we find that $[x, y]H(t) \in P$ for all $x, y, t \in \mathcal{R}$. Replacing y by yr, where $r \in \mathcal{R}$, we get $[x, y]rH(t) \in P$ for all $x, y, r, t \in \mathcal{R}$, which implies that by the primness of P that $H(\mathcal{R}) \subseteq P$.

Next, we consider the case $\mathcal{R} = B$, it follows that $d(F(y)) \in P$ for all $y \in \mathcal{R}$. It implies that for each $x, y \in \mathcal{R}$, we have $d([F(x), F(y)]) \in P$. Applying d to equation (2.13) and using the condition $d(P) \subseteq P$, we get

$$d(H([x, y])) \in P \text{ for all } x, y \in \mathcal{R}. \tag{2.18}$$

Replacing x by xy in (2.18) and using it, we find that

$$H([x, y])d(y) \in P \text{ for all } x, y \in \mathcal{R}.$$

Replacing x by xt and using it, we find $H([x, y])td(y) \in P$ for all $x, y, t \in \mathcal{R}$. Therefore, either $H([\mathcal{R}, \mathcal{R}]) \subseteq P$ or $d(\mathcal{R}) \subseteq P$. If $H([\mathcal{R}, \mathcal{R}]) \subseteq P$, then as in (2.17) we have $H(\mathcal{R}) \subseteq P$. Let's suppose that $d(\mathcal{R}) \subseteq P$, from (2.16) we have $(F(u)x - uF(x))[t, F(y)] \in P$ for all $x, y, t, u \in \mathcal{R}$, that means that $(F(u)x - uF(x)) \in P$ for all $x, u \in \mathcal{R}$ or $[t, F(y)] \in P$ for all $y, t \in \mathcal{R}$ (The second case is already discussed above). So, we assume that $F(u)x - uF(x) \in P$ for all $u, x \in \mathcal{R}$. Replacing u by uy, we get

$$\overline{F(u)yI_{\mathcal{R}}(x)} = \overline{I_{\mathcal{R}}(u)yF(x)} \text{ for all } x, y, u \in \mathcal{R},$$

where $I_{\mathcal{R}}$ is the identity map of $\mathcal{R}$.
Using Lemma 2.2, there exists $\overline{\lambda} \in \overline{C}$ such that $\overline{F(x)} = \overline{\lambda x}$ for all $x \in \mathcal{R}$. It implies that $F(x) - \lambda x \in P$ for all $x \in \mathcal{R}$. Hence,

$$[\{F - \lambda\}(x), \{F + \lambda\}(y)] \in P.$$

In view of our hypothesis, we get $(\lambda^2 + H)([x, y]) \in P$ for all $x, y \in \mathcal{R}$. □

In the same way, we can get the following result:

Theorem 2.7. *Let $\mathcal{R}$ be a ring and P a prime ideal of $\mathcal{R}$. If $\mathcal{R}$ admits a multiplier H and a generalized derivation F associated with a nonzero derivation d with $d(P) \subseteq P$, such that any one of the following assertions hold:*

(a) $F(x) \circ F(y) + H([x, y]) \in P$ *for all* $x, y \in \mathcal{R}$,
(b) $F(x) \circ F(y) + H(x \circ y) \in P$ *for all* $x, y \in \mathcal{R}$,
(c) $[F(x), F(y)] + H(x \circ y) \in P$ *for all* $x, y \in \mathcal{R}$,

then one of the following holds:

(i) $H(\mathcal{R}) \subseteq P$.
(ii) *There exists* $\lambda \in C$ *such that* $F - \lambda$ *maps* $\mathcal{R}$ *into* P *with* $(\lambda^2 + H)([x, y]) \in P$ *for all* $x, y \in \mathcal{R}$.
(iii) $\mathcal{R}/P$ *is a commutative ring.*

Proof. (a) By our assumption

$$F(x) \circ F(y) + H([x, y]) \in P \text{ for all } x, y \in \mathcal{R}.$$

Replacing x by xt in the above expression and using it, we conclude that

$$F(x)[t, F(y)] + (x \circ F(y))d(t) + x[d(t), F(y)] + H(x)[t, y] \in P. \tag{2.19}$$

Substituting ux for x in (2.19), we find that

$$\begin{aligned} F(u)x[t, F(y)] + ud(x)[t, F(y)] + u(x \circ F(y))d(t) - [u, F(y)]xd(t) \\ +ux[d(t), F(y)] + uH(x)[t, y] \in P \text{ for all } x, y, t, u \in \mathcal{R}. \end{aligned} \tag{2.20}$$

Left-multiplying (2.19) by u and comparing with (2.20), we get

$$\begin{aligned} (F(u)x - uF(x))[t, F(y)] + ud(x)[t, F(y)] \\ -[u, F(y)]xd(t) \in P \text{ for all } x, y, t, u \in \mathcal{R}. \end{aligned} \tag{2.21}$$

We process using the same approach as in Theorem 2.6, and finally we arrive at our result. We can reach the conclusions of (b) and (c) by using similar techniques as before, with the necessary variations of (c). $\square$

It is easy to prove that the maps $I_{\mathcal{R}}$ and $-I_{\mathcal{R}}$ are multipliers of $\mathcal{R}$. We get the following results by replacing H with $\mp I_{\mathcal{R}}$:

Corollary 2.8. *Let $\mathcal{R}$ be a ring and P be a proper prime ideal of $\mathcal{R}$. If $\mathcal{R}$ admits a generalized derivation F associated with a nonzero derivation d with $d(P) \subseteq P$, such that $\mathcal{R}$ satisfies one of the following assertions:*

(a) $[F(x), F(y)] \mp [x, y] \in P$ *for all* $x, y \in \mathcal{R}$,
(b) $F(x) \circ F(y) \mp [x, y] \in P$ *for all* $x, y \in \mathcal{R}$,

(c) $F(x) \circ F(y) \mp (x \circ y) \in P$ *for all* $x, y \in \mathcal{R}$,

(d) $[F(x), F(y)] \mp (x \circ y) \in P$ *for all* $x, y \in \mathcal{R}$, *then one of the following holds:*

(i) *there exists* $\lambda \in C$ *such that* $F - \lambda$ *maps* $\mathcal{R}$ *into* P *with* $(\lambda^2 \mp I)([x, y]) \in P$ *for all* $x, y \in \mathcal{R}$;

(ii) $\mathcal{R}/P$ *is a commutative ring.*

Replacing H by $-I_{\mathcal{R}}$ in the Theorem 2.6 and P by $\{0\}$, we get the following corollary:

Corollary 2.9. *If* $\mathcal{R}$ *is a prime ring admitting a strong commutativity preserving (SCP) generalized derivation* F *associated with a nonzero derivation* d, *then one of the following assertions holds:*

(1) *There exists* $\lambda \in C$ *such that* $F(x) = \lambda x$ *for all* $x \in \mathcal{R}$ *with* $\lambda^2 = 1$;

(2) $\mathcal{R}$ *is a commutative ring.*

Replacing H by $-I_{\mathcal{R}}$ in the theorem 2.7 and P by $\{0\}$, we obtain the following corollary:

Corollary 2.10. *Let* $\mathcal{R}$ *be a prime ring. If* $\mathcal{R}$ *admits a generalized derivation* F *associated with a nonzero derivation* d, *such that any one of the following assertions hold:*

(a) $F(x) \circ F(y) = [x, y]$ *for all* $x, y \in \mathcal{R}$,

(b) $F(x) \circ F(y) = x \circ y$ *for all* $x, y \in \mathcal{R}$,

(c) $[F(x), F(y)] = x \circ y$ *for all* $x, y \in \mathcal{R}$,

then one of the following holds:

(i) *There exists* $\lambda \in C$ *such that* $F(x) = \lambda x$ *for all* $x \in \mathcal{R}$ *with* $\lambda^2 = 1$;

(ii) $\mathcal{R}$ *is a commutative ring.*

References

1. A. Ali, M. Yasen and M. Anwar, *Strong commutativity preserving mappings on semiprime rings*, Bull. Korean Math. Soc. **4** (43) (2006), 711-713.
2. C. K. Liu and P. K. Liau, *Strong commutativity preserving generalized derivations on Lie ideals, Linear Multilinear Algebra*, **59** (2011), 905-915.
3. C. K. Liu, *Strong commutativity preserving generalized derivations on right ideals*, Monatsh. Math. **166** (2012), 453-465.
4. E. C. Posner, *Derivations in prime rings*, Proc. Amer. Math. Soc. **8** (1957), 1093-1100.
5. F. A. A. Alhmadi, A. Mamouni and M. Tamekkante, *A generalization of Posner's theorem on derivations in rings*, Indian J. Pure Appl. Math. (51) **5** (2020), 187-194.
6. H. E. Bell and M. N. Daif, *On commutativity and strong commutativity preserving maps*, Canad. Math. Bull. (37) **4** (1994), 443–447. doi: 10.4153/CMB-1994-064-x.

7. H. E. Bell and G. Mason, *On derivations in near rings and rings*, Math. J. Okayama Univ. **34** (1992), 135-144.
8. J. S. Lin and C. K. Liu, *Strong commutativity preserving maps in prime rings with involution*, Linear Algebra Appl. **432** (2010), 14-23.
9. J. Ma and X. W. Xu, *Strong commutativity-preserving generalized derivations on semiprime rings*, Acta Math. Sinica, (24) **11** (2008), 1835–1842. Doi: 10.1007/s10114-008-7445-0 (English Series).
10. J. S. Lin and C. K. Liu, *Strong commutativity preserving maps on Lie ideals*, Linear Algebra Appl. **428** (2008), 1601-1609.
11. K. I. Beidar, W. S. Martindale III and A. V. Mikhalev, *Rings with generalized identities*, Monographs and Textbooks in Pure and Applied Math. 196, New York: Marcel Dekker, Inc.(1996).
12. M. Brešar and C. R. Miers, *Strong commutativity preserving mappings of semiprime rings*, Canad. Math. Bull. **37** (1994) 457-460.
13. M. Brešar, *Commuting traces of biadditive mappings, commutativity preserving mappings and Lie mappings*, Trans. Am. Math. Soc. (335) **2** (1993), 525–546. doi: 10.1090/S0002-9947-1993-1069746-X.
14. M. Brešar, *Semiderivations of prime rings*, Proc. Amer. Math. Soc. (108) **4** (1990), 859–860.
15. M. Brešar, *On the distance of the composition of two derivations to the generalized derivations*, Glasg. Math. J. **33** (1991), 89-93.
16. M. S. Samman, *On strong commutativity-preserving maps*, Int. J. Math. Math. Sci. **6**(2005), 917–923. Doi: 10.1155/IJMMS.2005.917.
17. P. Semrl, *Commutativity preserving maps*, Linear Algebra Appl. **429** (2008), 1051-1070.
18. Q. Deng and M. Ashraf, *On strong commutativity preserving mappings*, Results Math. **30** (1996), 259-263.
19. T. K. Lee and T. L. Wong, *Nonadditive strong commutativity preserving maps*, Comm. Algebra, **40** (2012), 2213-2218.

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