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A GRAPH ASSOCIATED WITH MINIMAL IDEALS OF A RING

B. BARMAN AND K. K. RAJKHOWA *

Abstract. In this paper, a new kind of graph is introduced and investigated. The minimal ideal graph for a ring R with unity is an undirected graph whose vertex set contains all non-trivial ideals of R. We denote the graph by $mI(R)$ and the vertex set by $V(mI(R))$. Two vertices $P, Q \in V(mI(R))$ are adjacent if a minimal ideal p of R exists with $p \subset P$ and $p \subset Q$. We study the correlation of algebraic properties and graph theoretic properties of $mI(R)$. In this article, connectedness, diameter, clique number, chromatic number, regular character, cut vertex etc. are discussed.

1. Introduction

In recent times, the study of algebraic structures using graph theoretic tools attracts a lot of researchers. There are many correspondences which relate graph theory with ring theory. Some of them are seen in [1, 2, 5, 6, 7, 8, 16, 17, 18, 19]. In [7], Gaur and Sharma introduced the maximal graph of a commutative ring. They considered the vertex set as the elements of a commutative ring and any two vertices are adjacent if there exists a maximal ideal which contains both the vertices. The prime objective of their paper is to find the clique number, chromatic number of the graph and isomorphism classes of the corresponding ring. We, in this paper, define a graph, denoted by $mI(R)$, associated with the minimal ideals of a ring R with unity

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*Corresponding author .

and interpret the connectedness, clique number, chromatic number, independence number and domination number of $mI(R)$.

We recall certain ring theoretic and graph theoretic terminologies and notations that are needed in this sequel.

In this discussion, all graphs are simple and undirected. Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. For any two $x, y \in V(G)$, $\{x, y\} \in E(G)$ if and only if $x - y$ is a path in G. For $x \in V(G)$, $deg(x) = |\{y \in V(G) : \{x, y\} \in E(G)\}|$. A complete graph with n vertices is denoted by K_n. G is said to be r-regular if degree of each vertex of G is r. The length of shortest path between x and y is denoted by $d(x, y)$, and $d(x, y) = \infty$ if there exist no such paths. The diameter of G, denoted by $diam(G)$, is $max\{d(x, y) : x, y \in V(G)\}$, and the girth is the length of smallest cycle in G, denoted by $girth(G)$. The clique number, $\omega(G)$ is the number of vertices in the maximum complete subgraph of G. The chromatic number of G, denoted by $\chi(G)$, is the least number of colors to color all the vertices of G such that no two adjacent vertices receive the same color. An independence set is a set of mutually non adjacent vertices of G. We denote the independence number of G by $\alpha(G)$, which is the number of vertices in the largest independence set of G. A dominating set is a set of vertices such that any vertex of G is either a member of the set or is adjacent to a vertex of the set. The domination number is the cardinalty of the minimum dominating set of G and is denoted by $\gamma(G)$. If the vertex set of G can be partitioned in an independent set and a clique, then G is said to be a split. By $n(S)$, we mean the number of elements present in the set S.

The set of minimal ideals and the set of maximal ideals of a ring R are denoted by min_R and Max_R, respectively. A local ring contains exactly one maximal ideal. R is said to be a Gorstein local ring if and only if for any two ideals of R intersect non-trivially. Unless other wise specified, all rings are artinian and contain the unity element. The sum of all minimal ideals of R is denoted by $Soc(R)$.

2. Connectedness of $mI(R)$

Some properties related with connectedness, girth, diameter, completeness, regular character, cut vertex are studied in this section.

Remark 2.1. If R contains a unique minimal ideal, then $mI(R)$ is complete.

Remark 2.2. In $mI(R)$, an independent set is formed by the set of minimal ideals of R.

Remark 2.3. The graph $mI(F)$ is null if F is a field.

Theorem 2.4. *For a local ring R, $mI(R)$ is complete if and only if R is a Gorenstein ring.*

Proof. Let R be a Gorenstein ring. Consider any two vertices $S, T \in V(mI(R))$, then $S \cap T \neq 0$. As R is an artinian ring, so there exists some minimal ideal p of R with $p \subseteq S \cap T$, which implies $p \subseteq S$ and $p \subseteq Q$. Therefore S, T are adjacent. Hence $mI(R)$ is complete. Conversely, assume that $mI(R)$ is complete and P, Q are any two distinct vertices of $mI(R)$. Clearly P, Q are adjacent in $mI(R)$. So, there exists atleast one minimal ideal p such that $p \subseteq P$ and $p \subseteq Q$, which asserts that $0 \neq p \subseteq S \cap T$. Hence R is Gorenstein. $\square$

Theorem 2.5. *For an integral domain R, $mI(R)$ is complete.*

Proof. Let R be an integral domain and $I, J \in V(mI(R))$ be two non-trivial ideals. The there exist two non-zero element x, y such that $x \in I$ and $y \in J$, respectively. As R is an integral domain, $xy \neq 0$. Clearly, $xy \in < x >, < y >$ which implies $0 \neq xy \in < x > \cap < y > \subseteq I \cap J$. As R is artinian, there exists a minimal ideal m of R such that $m \subseteq I \cap J$, which infers $m \subseteq I$ and $m \subseteq J$. So, I, J are adjacent. Hence the result. $\square$

Remark 2.6. The converse of above theorem is not true, as Z_{16} is not an integral domain but $mI(Z_{16})$ is complete.

Theorem 2.7. *The graph $mI(R)$ is empty if and only if every ideal of R is minimal.*

Proof. If every ideal of R is minimal, then it is obvious that $mI(R)$ is empty. In opposite direction, let $mI(R)$ be empty and $I \in V(mI(R))$ be any vertex. If possible, we assume that I is not minimal. As R is artinian, there exist some minimal ideal m such that $m \subsetneq I$. Clearly I, J are adjacent. This contradiction affirms that every ideal of R is minimal. $\square$

Theorem 2.8. *The graph $mI(R)$ is disconnected if and only if R is a direct sum of two minimal ideals.*

Proof. Let $R = m_1 \bigoplus m_2$, where m_1 and m_2 are minimal ideals of R. Clearly, $m_1, m_2 \in V(mI(R))$ are two non adjacent vertices. If possible, consider that $mI(R)$ is connected. Then there exists some $I \in V(mI(R))$ such that I contains both m_1 and m_2. This implies that $R = m_1 + m_2 \subseteq I$, which is a contradiction. Therefore $mI(R)$ is disconnected. Conversely, let $mI(R)$ be disconnected. Then there exist atleast two vertices I, J, which are not connected. As R is artinian, there exists two distinct minimal ideals m_1 and m_2 contained in I and J, respectively. If $m_1+m_2 \neq R$, then $I-(m_1+m_2)-J$ is a path. This contradiction concludes that $m_1 \bigoplus m_2 = R$. Hence the theorem. □

Theorem 2.9. *If $mI(R)$ is disconnected for a commutative ring R, then it is empty.*

Proof. Assume that $mI(R)$ is disconnected. So, there exist at least two vertices I, J such that there is no path connecting them. Now, there exist two distinct minimal ideals m_1 and m_2 such that $m_1 \subseteq I$ and $m_2 \subseteq J$. Consider the ideal $m_1 + m_2$. Clearly $m_1 + m_2 = R$, otherwise $I - (m_1 + m_2) - J$ is a path. Again $\frac{R}{m_1} \cong m_2$ and $\frac{R}{m_2} \cong m_1$, which implies that m_1 and m_2 are maximal. Therefore $m_1 = I$ and $m_2 = J$. This asserts that any two disconnected ideals are minimal. Thus $mI(R)$ is empty. □

Theorem 2.10. *For a commutative ring R, $diam(mI(R)) = 1, 2, \infty$.*

Proof. If R is a direct sum of two minimal ideals, then $diam(mI(R)) = \infty$, by Theorem 2.8 and Theorem 2.9. If R is local Gorenstein ring, then $diam(mI(R)) = 1$, by Theorem 2.4. Consider that $mI(R)$ is connected and I, J are two non-adjacent vertices. Now there exist two distinct minimal ideals m_1 and m_2 of R which are contained in I and J, respectively. Take the ideal $m_1 + m_2$. If $m_1 + m_2 = R$, then $mI(R)$ becomes disconnected. So, $m_1 + m_2 \neq R$. This implies that $I - (m_1 + m_2) - J$ is a path. Therefore $diam(mI(R)) = 2$. □

Example 2.11. *Consider the ring $F = F_1 \times F_2 \times \cdots \times F_n$ with F_i's are fields. An ideal of F is of the form $L = \prod_{i=1}^{n} P_i$, where $P_i = 0$ or F_i. The minimal ideals of F is of the form $q_k = \prod_{i=1}^{n} P_i$, where $P_i = 0$ for $i \neq k$ and $P_k = F_k$. So, F has n minimal ideals. Assume that S and T are any two ideals of R. The following two cases arise:*

Case-i: *Suppose there exists some q_k, $k \in \{1, 2, \ldots, n\}$ with $q_k \subseteq S, T$. Then we get the path $S - q_k - T$.*

Case-ii: *Assume that there does not exist any q_k, $k \in \{1, 2, \ldots, n\}$ with $q_k \subseteq S, T$. Consider $q_y \subseteq S$ and $q_z \subseteq T$, where $y, z \in \{1, 2, \ldots, n\}$. Take the ideal $D = \prod_{i=1}^{n} H_i$, where $P_i = 0$ for $i \neq y, z$ and $P_i = F_i$ for $i = y, z$. Thus, the path $S - D - T$ is present in $mI(F)$. Therefore, $diam(mI(F)) = 2$.*

Theorem 2.12. *If $mI(R)$ contains a cycle then $girth(mI(R)) = 3$.*

Proof. If R is a local Gorenstein ring, then $mI(R)$ is complete. So, $girth(mI(R)) = 3$. Notice that there exists at least one vertex $I \notin min(R)$, otherwise $mI(R)$ is empty. Consider two adjacent vertices I and J. If both of I and J are not minimal, then there there exists a minimal ideal m such that $m \subsetneqq I$ and $m \subsetneqq J$. Thus we get the cycle $I - J - m - I$, which implies that $girth(mI(R)) = 3$. If one of I and J is minimal, say I, then $I \subsetneqq J$. If R contains exactly one minimal ideal, then for any vertex $K(\neq I, J)$, we get $m \subsetneqq K$. This infers that $I - J - K - I$ is a cycle. If R contains more than one minimal ideal, then there exists at least one $I \neq m \in min(R)$. Clearly $m + I \neq R$, otherwise $mI(R)$ is empty. This implies that $(m+I) - J - I - (m+I)$ is a cycle. Therefore $girth(mI(R)) = 3$. □

Example 2.13. Consider the ring $F = F_1 \times F_2 \times \cdots \times F_n$ with F_i's are fields. An ideal of F is of the form $L = \prod_{i=1}^{n} P_i$, where $P_i = 0$ *or* F_i. Consider the ideal $X = \prod_{i=1}^{n} P_i$, where $P_i = F_i$ for $i = 1, 2$ and 0, otherwise; $Y = \prod_{i=1}^{n} P_i$, where $P_i = F_i$ for $i = 1, 3$ and otherwise 0, $Z = \prod_{i=1}^{n} P_i$, where $P_i = F_i$ for $i = 2, 3$ and otherwise 0. Also a minimal ideal of F is of the form $q_k = \prod_{i=1}^{n} P_i$, where $P_i = 0 \ for \ i \neq k$ and $P_k = F_k$. Thus, F has n minimal ideals. It is easily seen that $X - Y - Z - X$ is a cycle. This asserts that $girth(mI(F)) = 3$.

Theorem 2.14. *If S is a cut vertex of $mI(R)$, then $K = m_1 + m_2$, for some $m_1, m_2 \in min(R)$.*

Proof. Observe that if $mI(R)$ has a cut vertex, then $|min_R| \geq 2$. Let S be a cut vertex. Then $mI(R) \setminus \{S\}$ is disconnected. So, there exist two vertices P and Q such that S lies in every path joining P and Q. By Theorem 2.10, we find that $d(P, Q) = 2$. Therefore, $P - S - Q$ is a path. This implies that there must exist $m_1, m_2 \in min_R$ with $m_1 \subsetneqq P, S$ and $m_2 \subsetneqq Q, S$, respectively. If $m_1 = m_2$, then P and Q are adjacent, which is a contradiction. So, $m_1 \neq m_2$. Also, using Theorem 2.9, we get that $m_1 + m_2 \neq R$. Thus, we obtain the path $P - (m_1 + m_2) - Q$. Since S is cut vertex, therefore, $S = m_1 + m_2$. This completes the proof. □

Theorem 2.15. *The following holds for $mI(R)$:*

(i) *If I and J are two vertices of $mI(R)$ such that $I \subseteq J$, then $deg(I) \leq deg(J)$.*
(ii) *If $mI(R)$ is $r - regular$ graph, then $mI(R) = K_{r+1}$.*

Proof. (i) Let I and J be two vertices of $mI(R)$ such that $I \subseteq J$. Suppose K is any vertex which is adjacent to I. This gives that there must exist a minimal ideal m such that $m \subseteq I, K$. Since $I \subseteq J$, we get $m \subseteq J$. This implies that K and J are adjacent. Thus, we obtain that every ideal which is adjacent to I is also adjacent to J. Therefore, $deg(I) \leq deg(J)$.

(ii) Let $mI(R)$ be an $r - regular$ graph. Therefore, $deg(m_i) = r$, for each $m_i \in min_R$. Clearly, m_i is adjacent to each $m_i + m_j$, where $m_j \in min_R$ and $i \neq j$. This asserts that min_R is finite. Again, $deg(m_i + m_j) > deg(m_i)$, as m_j is adjacent to $m_i + m_j$, but not to m_i. Therefore, $|min_R| = 1$. This gives that $mI(R)$ is a complete graph with $r + 1$ vertices. Thus $mI(R) = K_{r+1}$. □

Theorem 2.16. *If $mI(R)$ is connected and $V(mI((R)) = min_R \cup max_R$, then $mI(R)$ is split.*

Proof. Take the subgraph of $mI(R)$ induced by max_R. Let $M_1, M_2 \in max_R$ with $M_1 \neq M_2$. If $M_1 \cap M_2 = 0$, then $\frac{R}{M_1} \cong M_2$ and $\frac{R}{M_2} \cong M_1$. This asserts that M_1 and M_2 are also minimal ideals, a contradiction to the fact that $mI(R)$ is connected. Thus, $M_1 \cap M_2 \neq 0$. It is obvious that $M_1 \cap M_2 \notin max_R$. Therefore, $P \cap Q \in min_R$. Hence, we obtain that the subgraph induced by max_R is complete. Again, by Remark 2.2, the subgraph induced by min_R is empty. Thus $mI(R)$ is a split. □

3. Clique number, chromatic number, independence number and domination number of $mI(R)$

In this section, we obtain some results related with coloring, clique number, chromatic number, independence number and domination number.

Theorem 3.1. *If R has two minimal ideals m_1 and m_2 with $deg(m_1) \geq deg(m_2)$, then $\omega(mI(R)) = \chi(mI(R)) = deg(m_1)$.*

Proof. Assume that S is the set of vertices that contains m_1 and Q is the the set of vertices that contains m_2, respectively. Then any two vertices of S are adjacent. Thus to color these vertices, we need $deg(m_1)$ different colors. Also, S forms a clique of $mI(R)$. Therefore, $\omega(mI(R)) \geq deg(m_1)$. Now, consider the set Q. Some of the vertices of Q are already colored, precisely those vertices that belong to $S \cap Q$. The set $Q \setminus S$ contains the uncolored vertices. Clearly, no vertex of S is adjacent to any vertex of $Q \setminus S$. Also, $n(Q \setminus S) = n(Q) - n(S \cap Q) = deg(m_2) - n(S \cap Q)$ and $n(S \setminus Q) = n(S) - n(S \cap Q) = deg(m_1) - n(S \cap Q)$. Since $deg(m_1) \geq deg(m_2)$, we get $n(Q \setminus S) \leq n(S \setminus Q)$. As no vertex of $Q \setminus S$ is adjacent to any vertex of $S \setminus Q$, therefore, the vertices of $Q \setminus S$ can be colored using the colors used in color the vertices in $S \setminus Q$. Thus, we obtain $\chi(mI(R)) = deg(m_1)$. Since $\omega(mI(R)) \leq \chi(mI(R))$, hence, $\omega(mI(R)) = \chi(mI(R)) = deg(m_1)$. The proof is complete. □

Theorem 3.2. *If R contains a finite number of minimal ideals $m_1, m_2, \dots, m_n$ such that $deg(m_1) \geq deg(m_2) \geq \cdots \geq deg(m_n)$, then $\omega(mI(R)) = deg(m_1)$.*

Proof. For $n = 1$, the result is obivious. For $n = 2$, the result is established in Theorem 3.1. Consider that the result is true for $n-1$, where $n \geq 3$. Take P_i to denote the set of vertices adjacent to m_i, $i \in \{1, 2, \dots, n\}$. Clearly, $\omega(mI(R)) \geq deg(m_1)$, as the elements of P_1 form a clique. Toward a contradiction, assume that $\omega(mI(R)) > deg(m_1)$. Then, there exists a clique S of $mI(R)$ containing more than $deg(m_1)$ elements. If $I, J \in S$, then there exists some $m_i \in min(R)$ such that $m_i \subseteq I, J$. If $I \in S$ contains m_n only, then I is adjacent to only those vertices which are adjacent to m_n. This implies that $|S| \leq deg(m_n) \leq deg(m_1)$, which is a contradiction. Thus I contains some $m_j \in min(R)$, $j \in \{1, 2, ..., n-1\}$. Now consider the ring $R^{'} = \frac{R}{m_n}$. Clearly, $R^{'}$ has $n-1$ minimal ideals. By assumption, $\omega(mI(R^{'})) = deg(m_1)$. For any $I \in S$, we have $m_n \nsubseteq I$, and therefore $\frac{S}{m_n}$ forms a clique of $mI(R^{'})$ and $|\frac{S}{m_n}| = |S|$. Thus, we obtain that $deg(m_1) = \omega(mI(R^{'})) \geq |\frac{S}{m_n}| = |S| > deg(m_1)$, which is a contradiction. Hence $\omega(mI(R)) = deg(m_1)$. □

Corollary 3.3. *$\omega(mI(R))$ is finite if and only if $deg(m_i)$ is finite for every $m_i \in min_R$, whenever $|min_R| < \infty$.*

Theorem 3.4. *If $Soc(R) \neq R$, then $\gamma(mI(R)) = 1, 2$.*

Proof. For a ring with unique minimal ideal, it is obvious that $\gamma(mI(R)) = 1$, by Remark 2.1. Let $|min_R| \geq 2$. Consider the set $D = \{m_1, \sum_{k \neq 1} m_k\}$. Assume that $u \in V(mI(R)) \setminus D$. It is clear that either $u - m_1$ or $u - \sum_{k \neq 1} m_k$ is a path in $mI(R)$. Thus, in this case $\gamma(mI(R)) = 2$. Hence, we conclude that $\gamma(mI(R)) = 1, 2$. □

Example 3.5. Consider the ring $R = R_1 \times R_2$, where R_i is an artinian ring for $i = 1, 2$. Any ideal of R is of the form $I = \prod_{i=1}^{2} K_i$, where K_i is an ideal of R_i . If q_i is a minimal ideal of R_i, then that of R are $q_1 \times (0)$ and $(0) \times q_2$, respectively. Consequently, the set $\{q_1 \times (0), (0) \times q_2\}$ is a dominating set. Hence, $\gamma(mI(R)) = 2$.

Example 3.6. Consider the ring $R = \prod_{i=1}^{n} R_i$, where R_i is an artinian ring and $i \in \{1, 2, \ldots, n\}$. Also, assume that q_i is a minimal ideal of R_i. Any ideal of R is of the form $I = \prod_{i=1}^{n} K_i$, where K_i is an ideal of R_i. Take $S = \prod_{i=1}^{n} K_i$, where $K_1 = q_1$, $K_i = 0$ for $i \neq 1$, and $T = \prod_{i=1}^{n} K_i$, where $K_1 = 0$, $K_i = q_i$ for $i \neq 1$. It is easy to observe that $\{S, T\}$ is a dominating set. Hence, $\gamma(mI(R)) = 2$.

Example 3.7. If F_i is a field for $i = 1, 2$, then $\gamma(mI((F_1 \times F_2)) = 2$.

Example 3.8. Consider the ring $F = \prod_{i=1}^{n} F_i$, where F_i is a field. An ideal of F is of the form $A = \prod_{i=1}^{n} K_i$, where $K_i = 0$ *or* F_i. Consider two ideals $S = \prod_{i=1}^{n} K_i$, where $K_1 = F_1$, $K_i = 0$ for $i \neq 1$, and $T = \prod_{i=1}^{n} K_i$, where $K_1 = 0$, $K_i = F_i$ for $i \neq 1$. It is easy to observe that the vertex not belonging to $\{S, T\}$ is adjacent to either S or T or both. Hence, $\gamma(mI(R)) = 2$.

Theorem 3.9. *If min_R is finite, then $\alpha(mI(R)) = |min_R|$.*

Proof. Assume that $min_R = \{m_1, m_2, m_3, \ldots, m_n\}$ is the set of minimal ideals of R. By Remark 2.1, min_R is an independent set. Therefore, $n \leq \alpha(mI(R))$. Consider the largest possible independent set $S = \{l_1, l_2, l_3, \ldots, l_p\}$. Thus, $\alpha(mI(R)) = p$. For each $l \in S$, there exists a $m_i \in min_R$ such that $m_i \subseteq l$. If $p > n$, then there exist at least two vertices $l_i, l_j \in S$ which contain the same minimal ideal. This implies that $l_i - l_j$ is a path, a contradiction to the fact that S is an independent set. Therefore $p = n$, that is $\alpha(mI(R)) = n$. Hence the result. □

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Bikash Barman
Department of Mathematics, Cotton University, Guwahati-781001, India
Email: barmanbikash685@gmail.com

Kukil Kalpa Rajkhowa
Department of Mathematics, Cotton University, Guwahati-781001, India
Email: kukilrajkhowa@yahoo.com